Fast Approximation of Probabilistic Frequent Closed Itemsets

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Outline

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Preliminaries

Preliminaries

Traditional Itemset Database

T

	a	b	c
t_0	x		x
t_1	x	x	x
t_2		x	
t_3		x	x

- We have a set of items $A = \{a_1, a_2, \ldots, a_m\}$
- We have a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$
- An itemset is any $I \subseteq A$
  - Ex. $I = \{a, b\}$
- Item is either present or not

T

	a	b	c
t_0	x		x
t_1	x	x	x
t_2		x	
t_3		x	x

- The *support* of an itemset I is the number of transactions the itemset occurs in database T, denoted as $Sup_T(I)$
  - Ex. $Sup_T(\{a, c\}) = 2$
- $Sup_{t_j}(I)$ is 1 if $I \subseteq t_j$ or 0 otherwise
- $Sup_T(I) = Sup_{t_0}(I) + Sup_{t_1}(I) + \cdots + Sup_{t_n}(I)$
- Any $I \subseteq A$ whose $Sup_T(I) \geq minsup$ is considered a *frequent* itemset

Uncertain Itemset Database

T

	a	b	c
t_0	0.9		0.21
t_1	0.45	1.0	0.34
t_2		0.88	
t_3		0.6	0.4

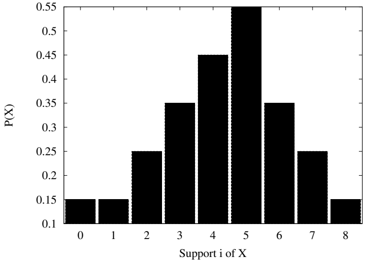
- Each item a has a probability of being in transaction t_j denoted as $Pr(a \in t_j)$
  - Ex. $Pr(a \in t_1) = 0.45$
- $Pr(I \subseteq t_j) = \prod_{a \in I} Pr(a \in t_j)$
  - Ex. $Pr(\{a, b\} \subseteq t_1) = Pr(a \in t_1) \cdot Pr(b \in t_1)$

In Uncertain Databases

- The probability that I occurs in a transaction t_j can be characterized as a Bernoulli random variable X_j^I with parameter $p = Pr(I \subseteq t_j)$
- If $X^I = \sum_{j=0}^{n} X_j^I$, then X^I is a random variable of the Poisson binomial distribution
- $Pr(X^I = i)$ is the probability the support of I is equal i

- Thus, the probability that the support of I is at least $i (X^I \geq i)$ is:

$$Pr(X^I \geq i) = \sum_{k=i}^{n} Pr(X^I = k)$$



- If $Pr(X^I \geq minsup) \geq \tau$, then I is considered a *probabilistic frequent itemset* (PFI) (Bernecker et al.)

Closed Itemset in Traditional Database

- More concise concise / much less redundant output
- If for all itemsets $I' \supset I$, $Sup_T(I') < Sup_T(I)$, then I is closed
- However, there is no concrete support of a uncertain itemset...but we do have the probability

- We defined the new concept of *probabilistic support* (Peiyi Tang et al., ACMSE 2011):

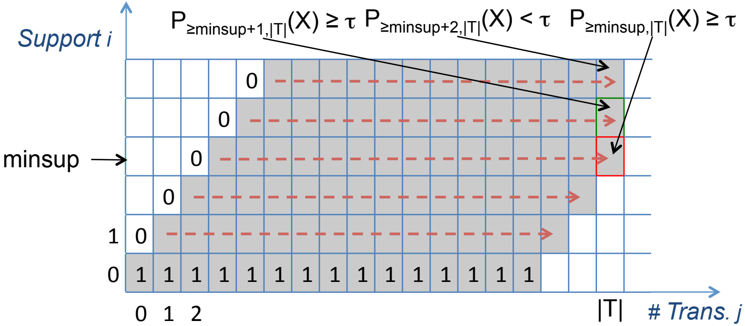
$$PS_T(I, \tau) = argmax_{i \in [0,n]}(Pr(X^I \geq i) \geq \tau)$$

Problem Statement: Probabilistic Frequent Closed Itemset (PFCI)

Given database T and user-defined thresholds τ and *minsup*, mine all itemsets I for which:

- I is probabilistically frequent, i.e. $Pr(X^I \geq minsup) \geq \tau$
- I is closed, i.e. for all $I' \supset I$, $PS_T(I', \tau) < PS_T(I, \tau)$

Each such itemset I we call a *probabilistic frequent closed itemset* (PFCI).



- A dynamic programming approach could be used to calculate $PS_T(I, \tau)$, i.e., with Bernecker et al.'s method
- This can be expensive, as to calculate $PS_T(I, \tau)$ one continues until $Pr(X^I \geq i) < \tau$

Approximating Probabilistic Frequent Closed Itemsets

Approximating Probabilistic Frequent Closed Itemsets

- Wang et al. showed that the Poisson binomial distribution can be approximated using the Poisson distribution
- The Poisson pmf is $Pr(X = i) \approx f(i, \mu) = \frac{\mu^i}{i!} \cdot e^{-\mu}$
  - Thus, the Poisson distribution cdf is $F(i, \mu) = \sum_{k=0}^{i} f(k, \mu)$
- We can use $\mu^I = \sum_{j=1}^{n} \prod_{a \in I} Pr(a \in t_j)$ — the expected support of I in T
- Let $Q(i, \mu^I) = 1 - F(i - 1, \mu^I)$, then $Pr(X^I \geq i) \approx Q(i, \mu^I)$
- $\widehat{PS_T(I, \tau)} = argmax_{i \in [0,n]}(Q(i - 1, \mu^I) \geq \tau)$

Problem Statement: Approx. Probabilistic Frequent Closed Itemset (A-PFCI)

Given an uncertain database T and user-defined threshold τ and *minsup*, mine all itemsets I for which:

- I is an approximate probabilistically frequent itemset, i.e. $\widehat{PS_T(I,\tau)} \geq minsup$
- I is closed, i.e. for all $I' \supset I$, $\widehat{PS_T(I',\tau)} < \widehat{PS_T(I,\tau)}$

Each such itemset is called an *approximate probabilistic frequent closed itemset* (A-PFCI)

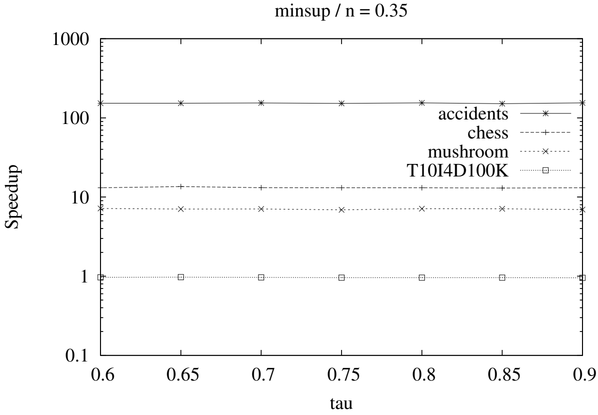
- Let μ_i ($i = 0, \ldots, n$) be the real numbers satisfying $Q(i, \mu_i) = \tau$.
  - i.e. $Q(0, \mu_0) = Q(1, \mu_1) = \cdots = Q(n, \mu_n) = \tau$
  - Because $Q(i, \mu)$ decreases with i and increases with μ: $\mu_0 < \mu_1 < \cdots < \mu_n$
- Using this fact, one can calculate $\widehat{PS(I, \tau)}$ for an itemset I as follows:
- If μ^I satisfies $\mu_i \leq \mu^I < \mu_{i+1}$ for an $i \in [0, n]$, then we have:
  - $\tau = Q(i, \mu_i) \leq Q(i, \mu^I)$
- In addition, we also have the following—for the same reason:
  - $Q(i+1, \mu^I) < Q(i+1, \mu_{i+1}) = \tau$
- This shows that i is the largest value such that $Q(i, \mu^I) \geq \tau$.

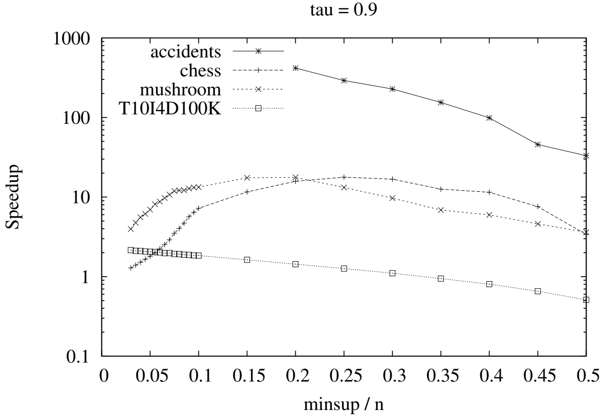
```
function CalcApproxProbSup(itemset I)
    float μ^I ← 0;
    foreach transaction j ∈ T do
        float product ← 1;
        foreach a ∈ I do
            product ← product · T[j][a];
        end foreach
        μ^I ← μ^I + product;
    end foreach
    if μ^I < μ_minsup then
        return −1
    else
        for i = minsup + 1 to n do
            if μ^I < μ_i then
                return i − 1;
            end if
        end for
    end if
end function
```

- Using this method, to calculate $\widehat{PS_T(I,\tau)}$ we need only to "lookup" the right value using the precomputed μ_i $(i = minsup + 1, \ldots, n)$

Experimental Evaluation

Experimental Evaluation


minsup / n = 0.35
1000
100
10
1
0.1
Speedup
0.6
0.65
0.7
0.75
0.8
0.85
0.9
tau
accidents
chess
mushroom
T10I4D100K


tau = 0.9
accidents
chess
mushroom
T10I4D100K
Speedup
minsup / n
1000
100
10
1
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Conclusions

Conclusions

- We define the new concept of an *approximate probabilistic frequent closed itemset* (A-PFCI)
- Will decrease the redundancy and size of output
- Developed an algorithm to mine these new concepts called A-PFCIM

Thank You
Questions?

paper / slides / code
website: erichpeterson.com